RESEARCH ARTICLE

Comparative Analysis on Interpolation Methods for Bathymetric Data Gaps

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Abstract

Light Detection and Ranging (LIDAR) technology delivers high accuracy elevation values and ground features. However, the capability of this technology is inhibited in terms of its strength to penetrate certain surfaces. For instance, LIDAR is limited to the elevation values of the river water surface and not the elevation of its riverbed. Hence, topographic and bathymetric surveys are conducted to obtain an accurate set of elevation values for areas where the technology is unable to permeate. Bathymetric surveys are conducted using a scientific echo sounder equipment, which utilizes sonar technology to determine the river depth relative to the water's surface by transmitting sound pulses and calculating the interval between the emanation and regress of a pulse per unit time. Like in all remote sensing measurements, errors are inevitable. Noise and external factors that cause faulty or bad readings result in data gaps. Gaps in the gathered elevation data sets can only be identified during filtering, which is done after the actual survey. In addition, covering the gaps back in the field would mean additional costs. This study aims to maximize data gathered by using different interpolation methods to generate points in the data gaps. Inverse Distance Weighting (IDW), Spline, and Kriging methods are used to extrapolate the values within the gaps. These values are then used together with the rest of the data for bathymetric data integration into the LIDAR data using IDW. Statistical calculations are shown to analyze the accuracy and efficiency of the results.

Keywords: bathymetry • interpolation • remote sensing limitations

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Author Contribution: JEC, VBC: conceptualization, writing - review and editing, methodology, supervision, project administration, resources, and validation; ZJID, CLAL, TMNL, DAT: formal analysis, writing - original draft; CLAL, TMNL, DAT: methodology, software, visualization; VBC: software; JEC: funding acquisition

Editor: Emma Ruth V. Bayogan, PhD, University of the Philippines Mindanao, PHILIPPINES

Received: 14 May 2020 **Accepted:** 28 December 2021 **Published:** 31 December 2021

Funding Source: The study was made possible by the Department of Science and Technology – Philippine Council for Industry, Energy, and Emerging Technology Research and Development through the Geo-SAFER Project.

Competing Interest: The authors have declared no competing interest.

Citation: Acosta JE, Calag VB, Diche ZJI, Lazaga CLA, Lopena TMN, Tatsuda DA. 2021. Comparative analysis on interpolation methods for bathymetric data gaps. Banwa B 16: art058.

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Introduction

Raw LIDAR-derived elevation models have high accuracy elevation z-values for terrain and surface models. However, the considered elevation values are limited to the above water level measurement of topographical features. To enhance the capability of this technology to provide as much information as possible, geodetic leveling activities are conducted. To obtain elevation values or underwater depth and map certain underwater features, bathymetric surveys are carried out.

Hydrographic and bathymetric surveys are necessary for various kinds of research studies, such as scour and stabilization, flood inundation and mapping, spill and fill, and others. Various bathymetric survey techniques with corresponding survey-grade equipment sets are employed for different purposes of hydrographic measurements.

The data obtained from these hydrographic surveys are then integrated into the elevation models to hydrologically correct the depth of the specific water body. Similar to the variation of techniques employed during bathymetric survey, there is also a selection of bathymetric interpolation methods to cater to the variation of bathymetric survey methods, bathymetric point density, and various types of water bodies. The integration of both elevation datasets in topography and bathymetry is vital to the completion of the digital elevation model.

This study compares several interpolation methods of river bathymetry data by studying the calibration bathymetry points and calculating the root mean square error of the generated bathymetry elevation model to the validation bathymetry points through the means of creating a data gap. This data gap was created manually from an otherwise continuous stream of bathymetric data. The interpolation methods were tested on how well they filled these gaps. In particular, the study utilized three interpolation methods to generate values: Inverse Distance Weighting (IDW), Kriging, and Spline. Each method has its own set of configurations that are explained further in the paper. The best configuration for each method was used together with the rest of the data for bathymetric data integration using IDW, of which the results were validated and compared against each other.

This study dealt with a comparison among spatial interpolation methods for computing elevation or z-values in data gaps of bathymetric data used to measure the elevation of the riverbed in meters above sea level (MASL), then integrated to the light detection and ranging (LIDAR) derived digital elevation model, which has a 1-meter resolution, to rectify the elevation of the riverbed by interpolating the topographic surface with the elevation values obtained by survey-grade equipment. Manual surveys are conducted to gather bathymetric data that represents the riverbed elevation values, which can be randomly distributed and may sometimes carry erratic values and/or not carry sensible information at all due to instrument limitations. Hence, interpolation of the values of the points in the necessary segments is performed to predict the missing values, in lieu of its error or absence, using the neighboring sets of points.

The objective of this study was to outline and compare different kinds of interpolation methods and their varying configurations to get closer to filling in missing data.

Review of Related Literature

Digital Elevation Model

The LIDAR-derived digital elevation model (DEM) is classified into two models—the digital terrain model (DTM) and the digital surface model (DSM), which showcases different topographical features. DTM refers to the topographic configuration of the featureless, bare Earth (Chen et al. 2017). DSM contains elevation values of the features found on the surface of the Earth, including both man-made and natural objects (Chen et al. 2017; Maune et al. 2007). Using a matrix structure in a raster (grid) format, elevation values, and topological relations between points in grid cells are recorded to form the DEM (Ramirez 2006). The resolution of a grid DEM is equivalent to the grid size of the DEM, which reflects the ground distance (Liu 2008).

Bathymetry

Hydrographic surveys are conducted to acquire data from water surfaces. These hydrographic surveys focus on the measurement and data collection of the bottom of any form of waterbody, such as oceans, lakes, and rivers (NOAA 1976). Bathymetric surveys are directed towards obtaining data, specifically, the elevation of the body of water's surface. In the case of this study, this refers to the elevation of the riverbed. Bathymetric surveys are customarily performed using acoustic echo sounder equipment, which can return accurate depth profiles (Gao 2009) calculated from the interval between the return times of the pulses on the surface (Klemas 2011). However, the acoustic echo sounder equipment has certain limitations. It is limited by efficiency and accessibility (Gao 2009), and acquisition of bathymetric data by this piece of equipment on shoal waters pose difficulties due to certain environmental conditions and technical considerations (Tronvig 2005).

Interpolation

Interpolation is a mathematical process of approximation, which determines a set of values for parameters or points given the values of its neighboring data (Mitas and Mitasova 2005). In the geographic information systems (GIS) environment, interpolation methods are programmed to predict values given a set of discrete or continuous data. Interpolation has practical uses in data management, specifically known data alongside missing data, where long-term cycles are known (Kaya 2014). However, no specific interpolation methods are strictly prescribed for use on bathymetric data (Curtarelli et al. 2015).

Inverse distance weighting

The inverse distance weighting (IDW) interpolation method is a local neighborhood approach that makes use of the values of its nearest neighbors by distance to derive a set of neighboring values (Watson and Philip 1985). IDW builds its basis on the premise that values at the data gaps (unsampled locations) can be estimated using the weighted average values of the points at a certain neighboring distance (Mitas and Mitasova 2005), given that these weights are inversely proportional to a given distance (Watson 1992). IDW is calculated as:

$$Z_j = \frac{\sum_{i=1}^{n} \frac{Z_i}{(h_{ij}+\delta)^{\beta}}}{\sum_{i=1}^{n} \frac{1}{(h_{ij}+\delta)^{\beta}}} \tag{1}$$

where Z_j is the unknown value to be interpolated, Z_i is the known value, β is the weight, δ is the smoothing factor, and h_{ij} is the separation distance, calculated as:

$$h_{ij} = \sqrt{(\Delta x)^2 + (\Delta y)^2} \tag{2}$$

where Δx and Δy are the distances between the unknown point j and the known point i according to reference axes (Mitas and Mitasova 2005).

Using IDW interpolation in ArcMap requires the power variable. Its purpose is to determine the influence of the sample points used to determine the value. Its value must be greater than zero. The higher the power, the more influence the nearest points have on the value (University of Namur Department of Geography 2003).

Kriging

Kriging interpolation is a geostatistical approach that takes the values of neighboring points and their respective locations as basis for estimation of the values of the specific points at a location (Kiš 2016; Longley et al. 2010) primarily revolving around the principle that point values near sampled locations should be assigned a greater weight in approximating the values for prediction in unsampled locations to improve its accuracy (Kiš 2016), with the assumption that the distance (with respect to the location) between point values in sampled locations have a spatial correlation that can be a basis for the variation of the surface (Childs 2004). The Kriging interpolation method was originally designed for approximations in the mining industry (Tang 2005), developed by Georges Matheron and Daniel Krige, with principle on the theory of regionalized variables (Kerry and Hawick 2005). Ordinary Kriging (OK) is a commonly utilized Kriging method and is referred to as the best linear unbiased estimator (Kiš 2016). The Kriging algorithm is expressed as:

$$Z(S_0) = \sum_{i=1}^{n} \lambda_i Z(S_i) \tag{3}$$

where n is the number of values, λ_i is the weight for the measured value at the i^{th} location, and S_0 represents the location of the value for prediction.

Spline

Spline interpolation is a piece-wise polynomial interpolation, which approximates values by using mathematical functions and splines to fit values into several fixed points with values (Ikechukwu et al. 2017) while minimizing the curvature of interpolated surface (Childs 2004). In comparison to IDW, the Spline interpolation method is designed to consider point values outside a minimum-maximum range of values in the sample data during the process of estimation (Liu 2008), which carries the advantage of this method in predicting values in ridges and valleys (Childs 2004). Spline interpolation algorithm is represented as:

$$S(x,y) = T(x,y) + \sum_{j=1}^{n} \lambda_j R(r_j) \tag{4}$$

where n is the number of points, λ_j are coefficients found by the solution of a system of linear equations, and r_j is the distance from (x, y) to (x_j, y_j).

There are two types of Spline interpolation in ArcMap: Spline and Spline with Barriers. Spline only requires the weight factor, which has to be greater than zero. At the minimum 0.1, the Spline interpolation method will try to closely match the data. The greater value the weight factor has, the smoother the fit will be (Smith 2015). The other Spline interpolation method, Spline with Barriers, utilizes breaklines in order to constrain the influence of closer points that are considered coincident points (ArcGIS 2016).

Methodology

Study Area

The bathymetric dataset used for this study is the length of the Dapnan River located in the Municipality of Baganga, Davao Oriental, Philippines. The bathymetric points were obtained using South S86 and Trimble Survey Grade Global Navigation Satellite System (GNSS) receivers, in a combination of zigzag, cross-section, and centerline manner along the length of the river with a total stretch of approximately 22 km. This was gathered by the Data Validation Team of the Geo-SAFER Southeastern Mindanao Project (2017-2019) of the University of the Philippines Mindanao.

To conduct the study, a continuous stream of actual bathymetric data was selected as a baseline for comparison. The chosen study area is a portion of Dapnan River with ideal conditions for the baseline, with a length of 1 km. The survey path covers 1.29306 km, traversing the center as well as some embankments in a zigzag manner. The area is home to an uninterrupted stream of 213 sample data points. Figure 1 shows this configuration, superimposed to the LIDAR digital terrain model of the area.

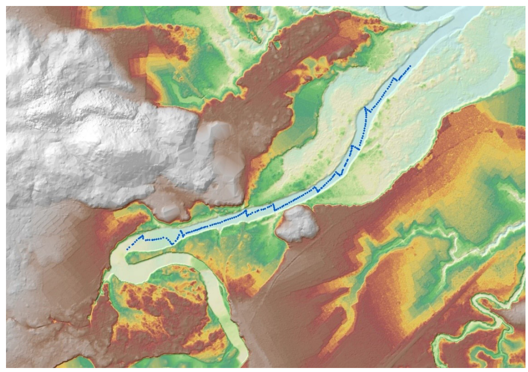

FIGURE 1 The 213-point bathymetric dataset superimposed to the topographic dataset, with the continuous 51-point artificial data gap area pointed out by a black brace

Topography and Bathymetry Dataset

The DTM used for integration with the combination of surveyed and interpolated bathymetric data was acquired using LIDAR technology, with a 1-meter by 1-meter resolution, in 2014 by the Data Acquisition Component of the Disaster and Risk Exposure Assessment for Mitigation (DREAM) Program.

Data Gap

From the 213 data points of the bathymetric dataset, 51 points were intentionally selected as missing values and served as data gap. These 51 continuous points were situated right in the middle of the sampled location, with 81 points before it and another 81 points after. This was made to ensure that there are sufficient value samples for each side of the gap as the basis for the prediction of values in the data gap during the application of the different interpolation methods and algorithms.

Interpolation Methods

Three interpolation methods were used to extrapolate the values for the gap. The best results for each method were determined based on how close these were to the actual values. These best results were then used in bathymetric data integration. Table 1 shows the common base parameters required for all the methods. IDW method used five (5) different values in the power parameter: 0.25, 0.5, 2, 3, and 6, which are then done for both With Barriers and Without Barriers techniques, totaling to a combination of ten (10) configurations. The Kriging method was implemented in five different semivariogram models, namely: spherical, circular, exponential, Gaussian, and linear. Spline method was implemented using both Regularized and Spline with Barriers techniques with values: 0.1, 0.5, and 1, totaling to a combination of six (6) configurations. The parameter with the smallest root mean square errors (RMSE) for each method is then chosen for bathymetric interpolation. Figure 2 sums these steps up in a flowchart.

TABLE 1 Common parameters present in all methods

Parameter	Constant value
Cell size	1
Number of points	12
Search radius (for Inverse Distance Weighting and Kringing methods)	Variable

Comparison of Interpolation Methods

Using the best results from the interpolation methods, each result was integrated into the LIDAR DTM using IDW. A comparison is made by comparing their root mean square errors (RMSE) and standard deviation values. The sample bathymetric points, including those points with values derived from interpolation, were integrated to form an interpolated surface. The resulting interpolated surface of bathymetric data points were analyzed and compared in terms of RMSE, calculated as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2} \quad (5)$$

where n is the population and x is the deviation of the elevation values in comparison (Chai and Draxler 2014). The standard deviation values that are calculated for each configuration correspond to the typical change in values with the generated data points versus the actual data points. The lower this value is, the better.

Results and Discussion

The results were computed using the stated parameters in the Interpolation Methods section and presented with their results in Tables 2, 3, and 4 together with their root mean square errors (RMSE) and standard deviation. The With Barriers technique was observed to have improved the results for both Inverse Distance Weighting (IDW) and Spline methods. Figures 3 and 4 show the graphs of the trend line of IDW method. Figures 5 through 9 show line graphs of each semivariogram model of the Kriging method compared to the original dataset. Figure 10 shows the graphical representation of the interpolated values using Spline method.

Inverse Distance Weighting method results

Table 2 shows the results for the IDW method. The input of barriers helped produce results that were closer to that of the actual values. It also helped the power variable concentrate its influence over enclosed spaces, as opposed to the free-for-all that happened without the input of barriers.

Figures 3 and 4 show how the generated values were versus the actual values. It can be observed that the barriers (Figure 4) visually fit the actual values better than the IDW method without barriers (Figure 3), which itself produced a waterfall trend line that signified that it failed to smooth out the central values no matter the configuration. The configuration of IDW with Barriers at Power = 6 was chosen to represent IDW in the final step as it is the best performing one in terms of RMSE and standard deviation.

Kriging method results

Table 3 shows the results for the different semivariogram models available to the Kriging method. They all have closely low RMSE and standard deviation with the Gaussian Semivariogram Model as the worst-performing one of the five models.

Figures 5 to 9 show how the generated values from each Kriging Semivariogram Model

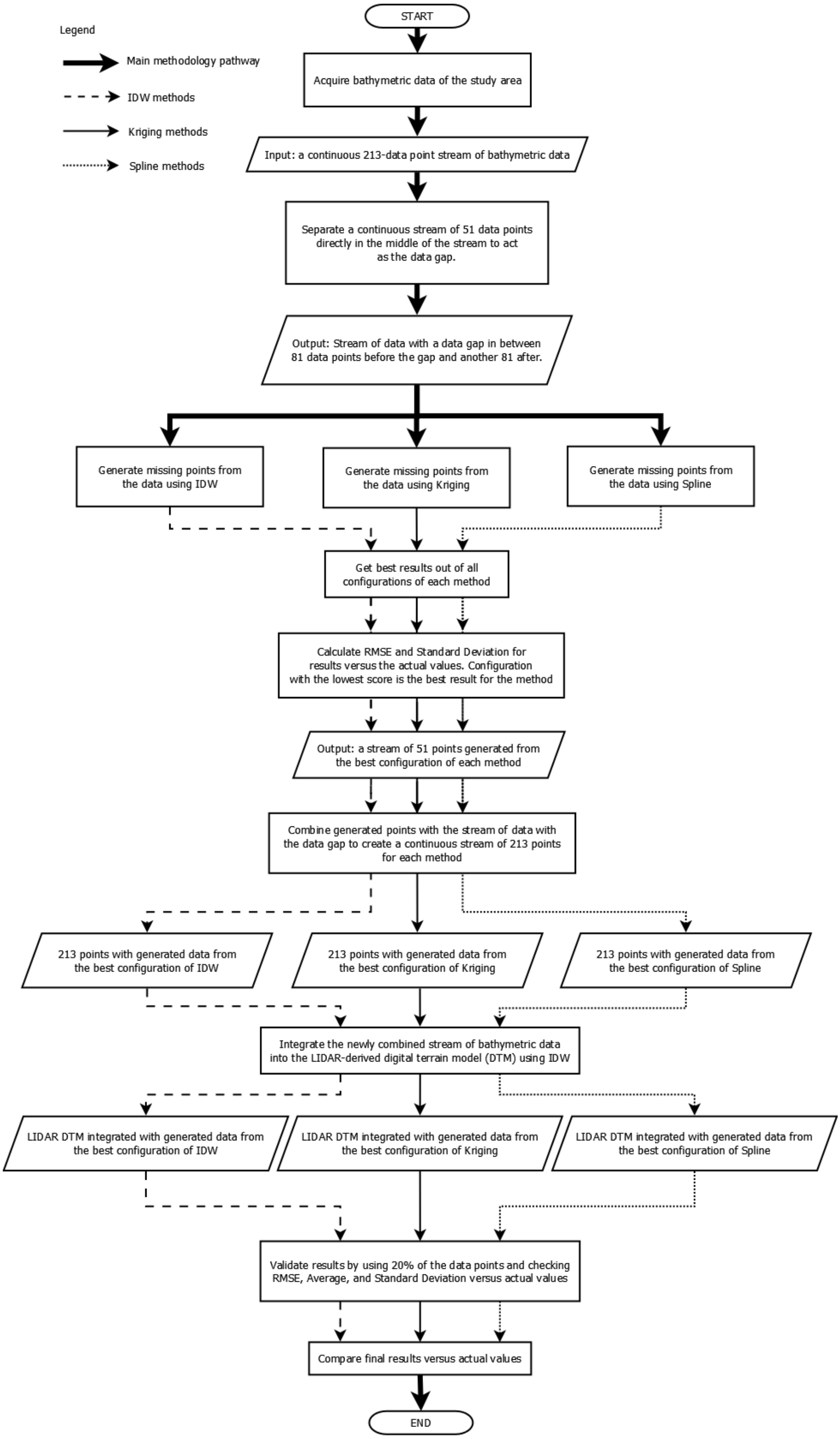

FIGURE 2 Flowchart of the study

configuration compare versus the actual values. Most configurations generated a very similar waterfall. The Gaussian model, however, looked visually better due to how it had some spikes similar to that of the actual values. While the Gaussian model has this advantage, it was not enough to justify its use as the best of the Kriging semivariogram models due to how bad it performs in terms of RMSE and standard deviation. Thus, the linear semivariogram model was used to represent Kriging. This configuration had the lowest RMSE and standard deviation values versus the actual values.

The visual peculiarity of the results for the Gaussian configuration may be hinting at this configuration to be chosen instead, but the data overall (RMSE, standard deviation) says otherwise as it is the worst performing configuration in the Kriging method.

TABLE 2 Parameters and results for Inverse Distance Weighting (IDW) method

Power	Without Barriers		With Barriers	
	Root mean square error	Standard deviation	Root mean square error	Standard deviation
0.25	0.45004	0.43700	0.40128	0.36770
0.5	0.44985	0.43622	0.36100	0.33262
2	0.44718	0.43101	0.27528	0.27357
3	0.44460	0.42766	0.26476	0.26705
6	0.42954	0.41193	0.25406	0.25399

TABLE 3 Parameters and results for Kriging method

Semivariogram model	Root mean square error	Standard deviation
Spherical	0.39040	0.31398
Circular	0.39035	0.31370
Exponential	0.39498	0.32073
Gaussian	0.44242	0.32147
Linear	0.39030	0.31342

Spline method results

Table 4 presents the results from the Spline method. As encountered in the IDW method, the input of barriers helped concentrate the algorithm into predicting values that are closer to the actual ones than the Spline method without it. The lower Spline value at 0.1, with barriers was chosen to represent the Spline method as it had the best results.

Figure 10 shows the performance of the different Spline configurations superimposed on the actual values. There is a huge difference between the regularized Spline (orange-themed lines) versus Spline with Barriers (blue-themed lines) values. The regularized configurations had the worst results of all the configurations so far and predicted values that were way off the range of the actual values. Configurations using Barriers, however, managed to stay within expected values although RMSE and standard deviation were mostly higher than those of the other two methods and their configurations.

The best performing configuration for the Spline method is the Spline with Barriers method at Smoothing factor 0.1. This means there was little smoothing done to produce the results.

The approximated values derived from the three interpolation methods applied to an identical dataset were incorporated into the existing values obtained from the ground survey. Table 5 outlines the results of each method's best configuration with RMSE, average, and standard deviation values for the validation points comprising 20% of the bathymetric data points. In each of the three interpolation methods, the best fit technique according to RMSE value, with respect to the individual parameter settings and models, are compared for analysis. Comparing the results, the parameters that will be used with the respective interpolation method are: (1) IDW with barriers at the 6^{th} Power, (2) linear semivariogram model for Kriging, (3) and Spline with barriers at 0.1 Smoothing Factor.

TABLE 4 Parameters and results for Spline method

Value	Regularized Spline (Weight)		Spline with barriers (Smoothing factor)	
	Root mean square errors	Standard deviation	Root mean square errors	Standard deviation
0.1	9.64757	6.94313	0.48298	0.35641
0.5	6.67182	6.57728	0.65400	0.39609
1.0	4.93911	4.97314	0.65526	0.39365

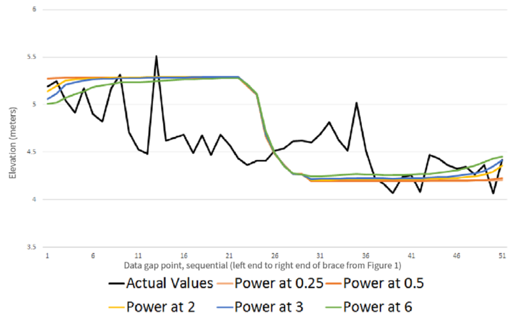

FIGURE 3 Inverse distance weighting interpolation method (without barriers)

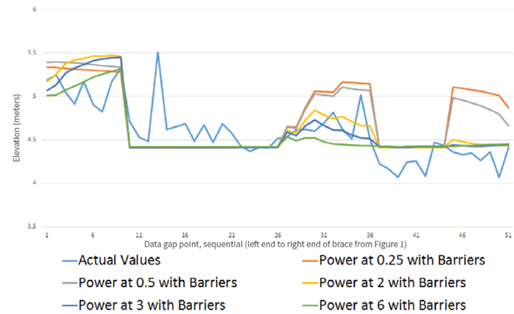

FIGURE 4 Inverse distance weighting interpolation method (with barriers)

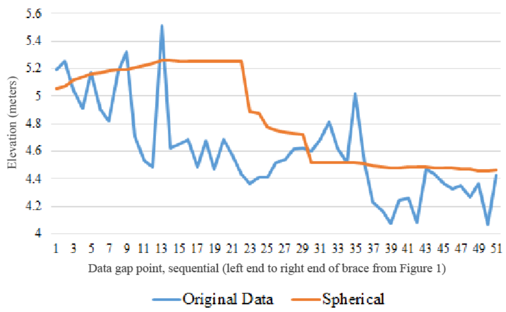

FIGURE 5 Kriging interpolation method (spherical semivariogram model)

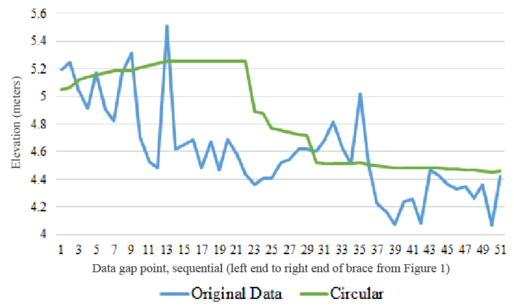

FIGURE 6 Kriging interpolation method (circular semivariogram model)

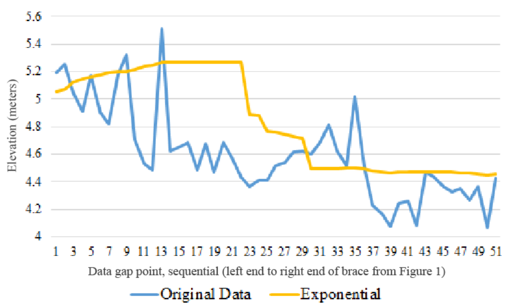

FIGURE 7 Kriging interpolation method (exponential semivariogram model)

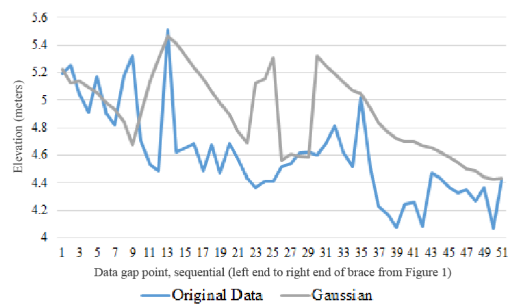

FIGURE 8 Kriging interpolation method (Gaussian semivariogram model)

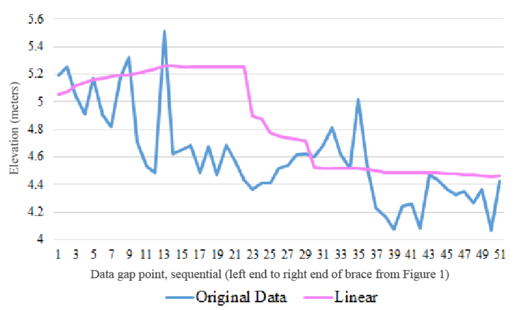

FIGURE 9 Kriging interpolation (linear semivariogram model)

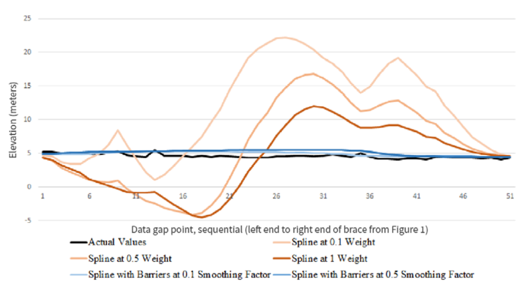

FIGURE 10 Spline interpolation method

TABLE 5 Validation results for each method

Method	Root mean square error	Average	Standard deviation
Inverse Distance Weighting with Barriers (Power: 6)	0.19764	-0.03543	0.19674
Spline with Barriers (Smoothing Factor: 0.1)	0.19669	-0.03633	0.19559
Ordinary Kriging (Linear semivariogram)	0.19646	-0.04266	0.19404

Conclusion and Recommendation

The bathymetric data integration of the combination of survey-acquired values and predicted values by interpolation techniques is feasible and necessary in the absence of values, considering mishaps during data gathering and field survey.

Results from each configuration in all the methods (Tables 2 to 4) are the main indicators as to how the generated datasets were versus the actual values. The figures of the trend lines for each configuration (Figures 3 to 10), however, paint a picture of the situation in its entirety. While configurations under the same method generally perform similarly, some of them can be indicators of a better fit for the data. This was most visible in the case of the Gaussian Kriging configuration (Figure 8), as well as methods that used Barriers for IDW (Figure 4) and Spline (Figure 10). Visual inspection of these trend lines will help in detecting behaviors in the data early in the process.

This study limited its comparison to three interpolation methods and findings showed that their performance is nearly similar (Table 5). Given the RMSE values, the average error computed is at 19 cm, which is within the range of acceptable RMSE of 20 cm. This similarity in their validation results can mean that there are many applicable ways to solve the data gap issue. However, the limitations of this study should be noted, and thus no configuration or method can be recommended as being the best one to fill data gaps. This situation may not be the same for other datasets or situations, but the methodology can be used to exhaust the many ways solving for a data gap can be achieved.

This study can be further developed by comparing the methods to different rivers with varying characteristics and considering other interpolation methods that are applicable to geospatial datasets, while modifying their parameter settings and observing their deviations as these parameters vary.

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